1. BASIC CONCEPTS

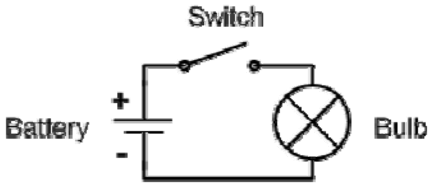


Fig: 1.1

The circuit in Fig. 1.1 is used to highlight some of the key aspects of a circuit.

a. A circuit is made up of elements, connected together by wire.
b. A circuit is represented on paper by a schematic diagram.
c. In order to draw the schematic diagram, we need circuit symbols to represent the elements used in the circuit.
d. The circuit in Fig. 1 contains a battery which is a voltage source.
e. When the switch is closed, current flows through the switch and the bulb.
f. There is a necessity for units to determine the performance of the circuit.

The questions that can be asked about the circuit in Fig. 1 are listed below.

a. What is the voltage of the battery ?
b. What is the current flowing through the circuit ?
c. What is the power delivered by the battery ?
d. What is the energy consumed by the bulb, say in one hour ?

To answer these questions, we need to know what the units for voltage, current, power and energy are.

The study of electric circuits is fundamental to study of electrical engineering. Even though the origin of circuit theory lies in electromagnetic field theory, it is easier to introduce electrical engineering via circuit theory, since it provides simple and sufficiently accurate solutions to practical electric systems. Circuit theory aims to predict and define the electrical behaviour of circuits, both qualitatively and quantitatively and its domain of application is very broad. Hence it is an area of study in its own right.

The study of circuit theory usually begins with definitions and axioms. Here we make a start with description of concepts in this chapter essentially at a superficial level. The purpose is to show the link between the electromagnetic field theory and the circuit theory.

BASIC UNITS
DERIVED QUANTITIES
CHARGE
CURRENT
ENERGY
VOLTAGE
WORK AND POWER
CIRCUIT VARIABLES
WORKED EXAMPLES
SUMMARY

1.1 BASIC UNITS

TABLE 1.1
The International System of Units

QUANTITY	UNIT	SYMBOL
Length	Metre	m
Mass	Kilogram	kg
Time	Second	s
Current	Ampere	A
Temperature	Degree kelvin	K
Luminous intensity	Candela	cd

Nowadays the International System of Units, known as ***SI*** system or the metric system, is used widely and the six basic units of this system are shown in Table 1.1. All other units are derived from these. For example, when mass of m kilograms is accelerated at rate a in m/s^2, the force behind is defined in newton and it is equal to $\boldsymbol{m} \times \boldsymbol{a}$. The basic units used most in this book are current and time.

One of the advantages of a metric system is that a system of decimal prefixes can be used with any unit, so that appropriate measures can be used as is warranted. The set of prefixes available is presented in Table 1.2.

TABLE 1.2
Standard Prefixes for SI Units

NAME OF PREFIX	SYMBOL	MULTIPLYING FACTOR
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

1.2 DERIVED QUANTITIES

TABLE 1.3
The International System of Units

QUANTITY	UNIT	SYMBOL	ABBREVIATED UNIT
Frequency	hertz	$\boldsymbol{f}$	Hz
Angular frequency	radian/second	Ω	rad/s
Phase Angle	radian or degree	θ, ϕ	rad or ^{o}
Power	watt	p	W
Energy	joule	w	J
Charge	coulomb	q	C
Voltage	volt	v	V
Impedance	ohm	$\mathbf{Z}$	Ω
Resistance	ohm	R	Ω
Reactance	ohm	X	Ω
Admittance	siemens	$\mathbf{Y}$	S
Conductance	siemens	G	S
Susceptance	siemens	B	S
Inductance, self	henry	L	H
Inductance, mutual	henry	M	H
Flux density	tesla	B	T
Flux	weber	ϕ	wb
Magneto-motive Force	ampere-turns	*mmf*	AT
Power Ratio	bel	$\log_{10}(p_2/p_1)$	B
Capacitance	farad	C	F

In this text, the derived quantities appear as the circuit variables. In this chapter, the discussion is limited to charge, current, voltage, power and energy. The other variables are described when they are used for the first time.

1.3 CHARGE

All electrical phenomena are manifestations of electric charge. The atomic particles of matter that carry charges are protons and electrons. Protons carry positive charges and electrons carry negative charges. The charge an electron carries is small and it is -1.602 x 10^{-19} coulomb(C). This means that 1 C contains 6.24 x 10^{18} electrons. In addition, this also means that charge exists in discrete quantities. Usually we use either the literal symbol Q or $q(t)$ when a reference to charge contained in a given quantity of matter is made. The ***conservation of charge*** is due to the basic physical postulate that a charge cannot be created or destroyed and that it can only be transferred. The transfer of charge leads to charge flow or current.

By nature, charge is bipolar, in the sense that there are both positive charges and negative charges. Separation of charges gives rise to voltage, or the electro-motive force, whereas charge in motion gives rise to current.

1.4 CURRENT

The *SI* unit for current is ***ampere***, and current is the rate of charge flow or the rate of change in charge. An ampere is equal to one coulomb per second. We can state that current $i(t)$ to be:

$$i(t) = \frac{dq}{dt} \,. \qquad (1.1)$$

From the above definition, we can find out the charge that passes through a surface in the time interval $-\infty$ to t. If the current through at any instant be $i(t)$ then

$$q(t) = \int_{-\infty}^{t} i(t) \cdot dt \qquad (1.2)$$

with the assumption that $q(-\infty) = 0$. In practice, an ammeter is used to measure current. In a laboratory, a current probe may be used in order to see the waveform of current on an oscilloscope.

WORKED EXAMPLE 1.1:

A copper wire carrying 10 A has a diameter of 1.5 mm. The flow of current is only due to electrons. The free electron concentration in copper is 10^{29} electrons per cubic metre. Find the average velocity of electrons in the wire.

SOLUTION:

The solution is obtained as follows. The current passing through the wire is known. First determine the charge passing through the wire per second. Then calculate the number of electrons that pass through the wire per second. Based on the number of electrons and the free electron concentration in copper, determine the volume of copper needed to hold the required number of electrons. Divide the volume by the area of the wire to get the average velocity of electrons in the wire.

$$\text{Charge / second flowing through the wire} = 10\ \text{C} = 6.24 \times 10^{19}\ \text{electrons}$$

$$\text{Volume of copper needed to hold 10 C} = \frac{6.24 \times 10^{19}}{10^{29}} = 6.25 \times 10^{-10}\ \text{m}^3$$

$$\text{Area of copper wire} = \frac{\pi}{4} \times 1.5^2 \times 10^{-6} = 1.767 \times 10^{-6}\ \ \text{m}^2$$

$$\text{Velocity of electrons} = \frac{\text{Volume}}{\text{Area}} = 3.537 \times 10^{-4}\ \text{m}$$

1.5 ENERGY

The ***law of conservation of energy*** postulates that energy cannot be created, but only transformed. Chemical energy, kinetic energy and atomic energy can be transformed into electrical energy. The reason for popularity of electric energy is its ease of transportability.

Any electric circuit obeys the law of conservation of energy. It means that the algebraic sum of power in a circuit is zero at any instant. If power associated with an element in a circuit is *p*, then

$$\sum p = 0 \qquad (1.3)$$

Examples that reflect equation (1.3) are provided in chapter 3, when Kirchoff's laws are presented.

1.6 VOLTAGE

If work is done on charge, the ratio of energy expended to charge is voltage. For example, work is done on charges by chemical reactions within a battery and the result is a voltage across its terminals. If we use literal *w* to indicate the work done on charge *q*, then

$$v(t) = \frac{dw}{dq}, \qquad (1.4)$$

where *w* is in joules, *q* is in coulombs and *v* is in volts. The ***SI*** unit for voltage is the volt, represented by literal V. In practice, a voltmeter is used to measure the voltage between two points.

1.7 WORK AND POWER

From equation (1.4) for voltage, we get that

$$dw = v.dq \qquad (1.5)$$

If the time taken for work *dw* be *dt*, then

$$\frac{dw}{dt} = v.\frac{dq}{dt} \qquad (1.6)$$

The rate of change of energy with respect to time is power. It is also known that current is the rate of charge, as defined by equation (1.1). Hence power *p* is:

$$p = \frac{dw}{dt} = v.\frac{dq}{dt} = v.i \qquad (1.7)$$

Electrical power is the product of voltage and current. The unit for power is watt(W).

1.8 WORKED EXAMPLES

WORKED EXAMPLE 1.2

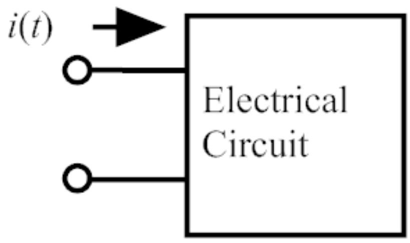


Fig: 1.2

The charge, *i*(*t*) flowing into the circuit, shown as a box in Fig. 1.2, is expressed to be:

$$i(t) = e^{-t} \cdot \sin(t), \quad \text{for } t \geq 0\,\text{s} \qquad (1.8)$$

Find

a. the total charge that flows into the circuit over the period $0 \leq t < \infty$, and the
b. the peak current and the instant at which it occurs.

SOLUTION:

Let the total charge flowing into the circuit be *Q*. Then

$$Q = \int_0^\infty e^{-t} \cdot \sin(t) \cdot dt = \left[-e^{-t} \cdot \cos(t)\right]_0^\infty + \int_0^\infty e^{-t} \cdot \cos(t) \cdot dt$$

$$= 1 + \left[\, e^{-t} \cdot \sin(t)\right]_0^\infty - \int_0^\infty e^{-t} \cdot \sin(t) \cdot dt = 1 - Q$$

$$\therefore Q = 0.5\,\text{C}$$

As shown above, it is necessary to make use of the formula for integration by parts. It is known that

$$d(u \cdot v) = u \cdot dv + v \cdot du$$

$$\int_0^\infty u \cdot dv = [u \cdot v]_0^\infty - \int_0^\infty v \cdot du$$

Alternatively, Euler's identity can be used to obtain Q.

$$\because e^{jt} = \cos(t) + j \cdot \sin(t),$$

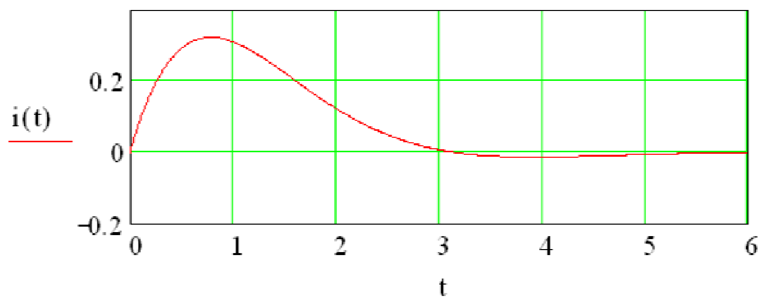
$$Q = \mathrm{Im}\left[\int_0^\infty \left(e^{-t} \cdot e^{jt}\right) \cdot dt\right] = \mathrm{Im}\left[\left(\frac{e^{-(1-j)t}}{j-1}\right)_0^\infty\right] = \mathrm{Im}\left[\frac{1}{1-j}\right] = \mathrm{Im}\left[\frac{1+j}{2}\right] = \frac{1}{2}\ \mathrm{C}$$

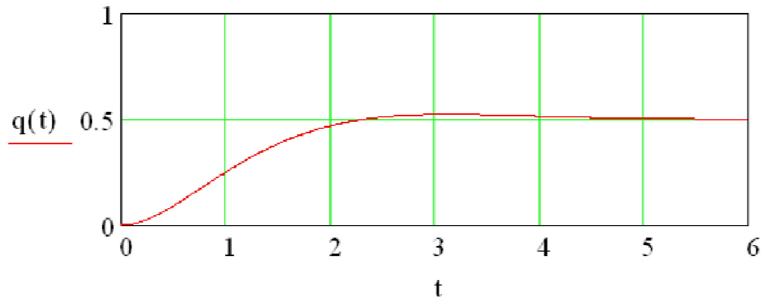
To find the peak current, differentiate current and equate the derivative to zero. Find the instant at which the derivative to zero

$$\frac{di(t)}{dt} = \frac{d}{dt}\left(e^{-t} \cdot \sin(t)\right) = e^{-t} \cdot \cos(t) - e^{-t} \cdot \sin(t)$$

$$\therefore\ \cos(t) - \sin(t) = 0,\ \ \tan(t) = 1,\ t = \frac{\pi}{4}\ \mathrm{s}$$

The peak value of current is 0.322 A and it occurs at t = 0.785 s.   The plots of current and charge are shown below.





WORKED EXAMPLE 1.3

The current, $i(t)$ flowing into the circuit, shown as a box in Fig. 1.2, is expressed to be:

$$i(t) = 1\ \mathrm{A\ for}\ t > 0\ \mathrm{s}$$

$$= 0\ \ \mathrm{A\ for}\ t \le 0\ \mathrm{s}$$

Sketch the waveform of $i(t)$ and $q(t)$.

SOLUTION:

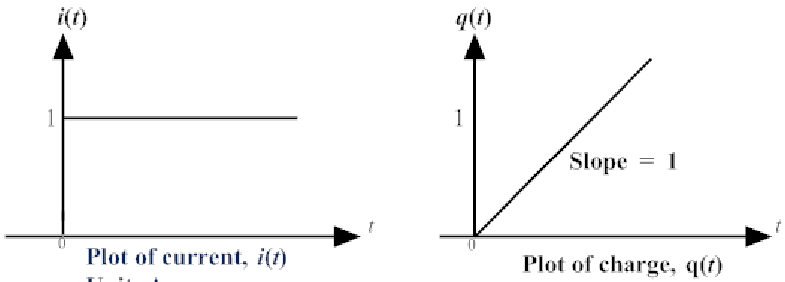


Fig. 1.3

$$\because i(t)=\frac{dq}{dt},\ q(t)=\int_0^t i(t)\cdot dt + q(0)$$

Assuming that $q(0) = 0$, we get that

$$q(t)=t \text{ C for } t>0 \text{ s}$$
$$=0 \text{ C for } t\leq 0 \text{ s}$$

The plots of $i(t)$ and $q(t)$ are presented by Fig. 1.3.

It is seen that the current has a discontinuity at $t = 0$ s and its value jumps at $t = 0$ s. Such a function is called as ***unit step function*** and is denoted by $u(t)$. Unit-step function has a value of unity for positive values of t and is zero for negative values of t. It is seen that the charge increases linearly with t when t is positive and is zero for negative values of t. Such a function is known as ***unit ramp function*** and is denoted by $r(t)$. In other words,

$$r(t)=t\cdot u(t)$$

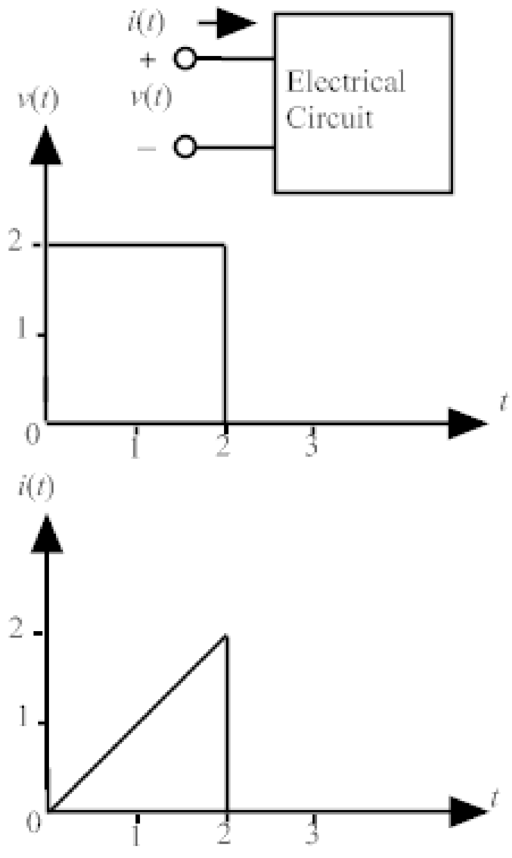
The expression for the unit ramp function shows how unit step function can be used for defining other functions.

WORKED EXAMPLE 1.4

The plots of voltage and supplied to a circuit are shown in Fig. 1.4. Show the plots of instantaneous power and the energy absorbed by the circuit.

SOLUTION:

From Fig. 1.4, the expressions for voltage and current are as follows:



Current and Voltage Supplied to a circuit

Fig: 1.4

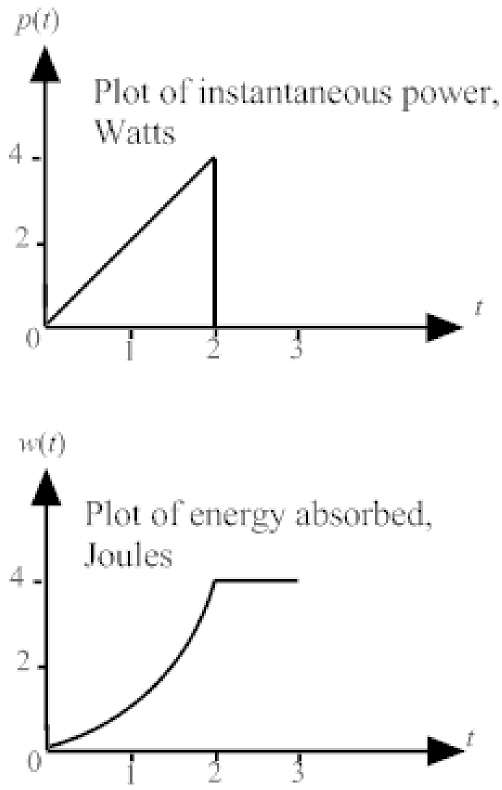


Fig: 1.5

$$v(t) = 2 \text{ V for } 0 \le t \le 2 \text{ s}$$
$$= 0 \text{ V for } t < 0 \text{ s}$$
$$= 0 \text{ V for } t > 2 \text{ s}$$
$$i(t) = t \text{ A for } 0 \le t \le 2 \text{ s}$$
$$= 0 \text{ A for } t < 0 \text{ s}$$
$$= 0 \text{ A for } t > 2 \text{ s}$$

Based on equation (1.7), we get that

$$p(t) = v(t) \cdot i(t)$$
$$= 2t \text{ W for } 0 \le t \le 2 \text{ s}$$
$$= 0 \text{ W for } t < 0 \text{ s}$$
$$= 0 \text{ W for } t > 2 \text{ s}$$

Energy is the integral of power, as can be understood from equation (1.7). Hence

$$w(t) = \int p(t) \cdot dt = \int_0^t v(t) \cdot i(t) \cdot dt$$

$$= t^2 \text{ J for } 0 \le t \le 2 \text{ s}$$

$$= 0 \text{ J for } t < 0 \text{ s}$$

$$= 4 \text{ J for } t > 2 \text{ s}$$

The plots of power and energy are shown in Fig. 1.5.

1.9 SUMMARY

This chapter has presented the fundamental units of the SI system. The other basic electrical parameters such as voltage, charge, power and energy have also been introduced.

EXERCISE PROBLEMS:

E1.1: Current, $i(t)$ flowing into a circuit is expressed to be:

$$i(t) = e^{-t} \cdot \left(t^2 + 2t\right) \text{for } t \ge 0 \text{ s}$$

a. Plot $i(t)$.
b. What is the peak value of $i(t)$ and when does it occur?
c. Find the total charge flowing into circuit.
d. Find the expression for current entering the circuit and plot it.

E1.2: Energy, $w(t)$ absorbed by a circuit is expressed to be:

$$w(t) = \frac{1}{3}t^3 + t^2 \text{ J for } 0 \le t \le 3 \text{ s}$$

a. Find the expression for power during the interval $0 \le t \le 3$ s.
b. Given that current, $i(t)$ is expressed as:

$$i(t) = t \text{ A for } 0 \le t \le 3 \text{ s}$$

$$= 0 \text{ A for } t < 0 \text{ s}$$

$$= 0 \text{ A for } t > 3 \text{ s}$$

find an expression for voltage $v(t)$.
c. Sketch the waveforms of $i(t)$ and $v(t)$.

E1.3 A laser generates 100 -mJ laser pulses of 25 ns duration.

a. What is the peak instantaneous power of the laser /

b. If 50 pulses are generated per second, what is the average power output of the laser ?

E1.4 A 12 V battery supplies 5 A for 1 hour and the charge supplied by the battery during this period amounts to one-tenth the AH rating of the battery. What is the AH rating of the battery?
