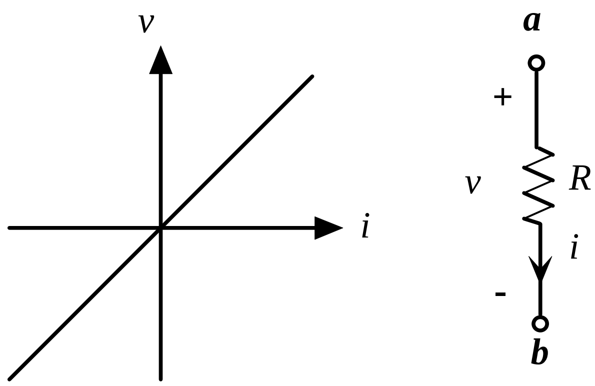
2. CIRCUIT ELEMENTS

2.1 INTRODUCTION

A circuit or a network is made up of elements, connected such that a meaningful operation results. The term, ***circuit,*** usually implies a closed path for current flow, whereas such an inference is not applicable to a network. It is possible to classify the elements used in circuits in a few ways. The common classification divides the elements into two types, called as passive or active. Sources and electronic devices are active elements, whereas the group of passive elements consists of resistor, inductor and capacitor. An element can also be classified to be linear or nonlinear, lumped or distributed, fixed or time-varying, and unilateral or bilateral. Further information on classification of elements is presented in another section.



Symbol of a resistor and its v-i characteristic

Fig. 2.1

2.2 PASSIVE ELEMENTS

Resistors, inductors and capacitors are called the passive elements. Here we assume in addition that they are linear, even though a real resistor tends to be slightly nonlinear.

2.3 RESISTOR

The v-i characteristic of a linear resistor is shown in Fig. 2.1. For a resistor,

$$v = R \times i \quad (2.1)$$

Equation (2.1) expresses the Ohm's law. The Ohm's law states that the voltage across a resistor varies proportionately with its current, where the constant of proportionality is its resistance, ***R***.

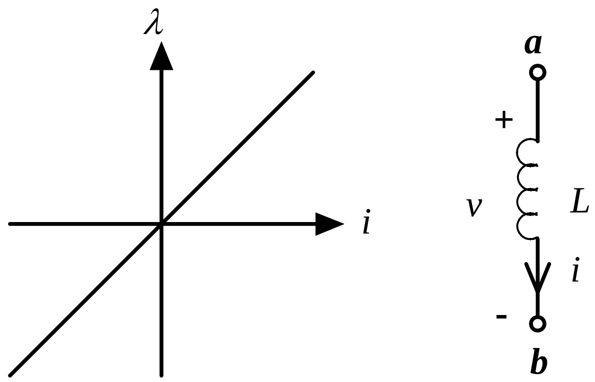
In other words, the ratio of voltage across a resistor to its current is its resistance in ohms, Ω. In an ideal resistor, the current through a resistor varies linearly or proportionately with the voltage across it and that is the reason why a resistor is said to be a linear element. Equation (2.1) also specifies how the polarity of voltage is related to the polarity of its current. From equation (2.1), it can be seen that the sign of voltage across a resistor is the same as that of its current. Let the terminals of a resistor be ***a*** and ***b***. Let current ***i*** enter the resistor via its terminal ***a*** and leave it via ***b***. Then the voltage at terminal ***a*** is positive with respect to that at ***b***, as shown in Fig. 2.1. **Equation (2.1) is said to describe the sign convention for passive elements, since power absorbed by a passive element is positive.** Since

$$p = v \times i \quad (2.2)$$

the sign convention used in equation (2.1) leads to the correct result. When ***v*** and ***i*** have the same sign, the product representing power absorbed is positive. From equation (2.1), it can be seen that both *v* and *i* must have the same sign, because physical resistance is always positive. The interpretation of the sign convention for a source is presented in the chapter on the Kirchoff's voltage law.

2.4 INDUCTOR

An inductor is an element which can store energy. It can also be called a dynamic element, since it stores energy. An inductor may have a magnetic core with or without air-gap and such an inductor may not be linear. On the other hand, an inductor with only an air-core is linear. In such an inductor, the relationship between the flux linkage and the current through it is linear, the constant of proportionality being its inductance, as shown in equation (2.3).



Symbol of an inductor and its λ-i characteristic

Fig. 2.2

$$\lambda = L(i) \times i \quad (2.3)$$

In the equation above, ***i*** is the current through the inductor at any instant, ***L(i)*** is the inductance and λ is the flux linkage in the inductor due to the current, where the flux linkage is the product of flux ϕ in the core and the number of turns N linked to flux. The unit for flux is Weber, whereas the unit for flux linkage is volt-second. Current flowing through a winding consisting of some turns of conductor wound around a core creates flux. In an iron-cored inductor, the inductance varies as a function of current. As the core tends to saturate, the value of inductance falls. But in an air-core inductor, the inductance is independent of current and is more or less a constant, depending on the geometry of the core and the number of turns in the inductor. **In a linear inductor, the flux linkage and the current vary proportionately, where the constant of proportionality is defined as inductance, *L*** The linear characteristic of an inductor is illustrated in Fig. 2.2.

By definition, the rate of change of flux linkage is voltage. When the inductance is independent of current through it, we can derive from equation (2.3) the following relationship between the voltage and the current of an inductor.

$$v = \frac{d\lambda}{dt} = \frac{d(L \cdot i)}{dt} = L \times \frac{di}{dt} \qquad (2.4)$$

It can be seen that flux linkage is the integral of voltage with respect to time and it is the reason why the unit for flux linkage is volt-second. From equation (2.4), we get that

$$\lambda(t) = \int_0^t v(t) \cdot dt + \lambda(0) \qquad (2.5)$$

where $\lambda(0)$ is the initial flux linkage.

The voltage across an inductor is positive when the current through it tends to increase. When the rate of rise of inductor current is positive, the voltage across inductor is positive. When the current through an inductor falls, the voltage across the inductor is negative. Note that the voltage across an inductor is dependent on the rate of change of its current. When an ideal inductor carries a fixed dc current, the rate of change of its current is zero and the voltage across the inductor is zero.

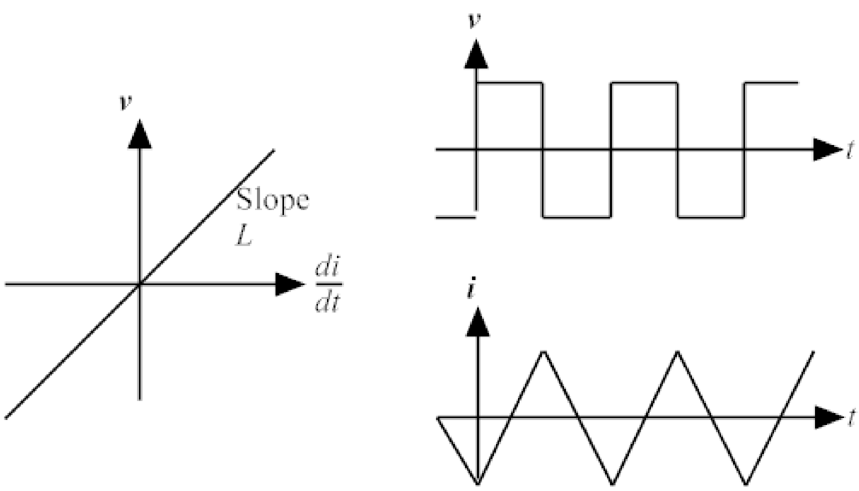


Fig. 2.3: v-i characteristic of an inductor

The v-i relationship of an inductor is illustrated in Fig. 2.3. If a square-wave signal is impressed on the inductor, the current is proportional to the integral of the voltage across the inductor. Conversely, if the waveshape of inductor current is triangular, the inductor voltage is made up of rectangular pulses.

When an inductor carries some current, there is energy stored in it. If the current through an inductor rises from zero to ***I*** in ***T*** seconds, the energy stored is calculated as follows. Let the energy stored be ***W*** joules. Then

$$W = \int_0^T \left[v(t) \times i(t) \right] \cdot dt \qquad (2.6)$$

Using equation (2.4), the above equation can be expressed as:

$$W = \int_0^T \left[L\frac{di}{dt} \times i(t) \right] \cdot dt = \int_0^I \left[L\, i(t) \right] \cdot di = \frac{1}{2} L \times I^2 \qquad (2.7)$$

It is known that energy is an integral of power with respect to time. Hence, the power absorbed by an inductor at a given instant determines the rate of change in energy stored inductor. If the energy stored in the inductor is to change suddenly, the source feeding the inductor has to supply inifnite power. Since physical sources have finite power, energy stored in an inductor or any system cannot change instantaneously. As change in energy stored is to occur due to a finite power, a finite time is required for that change. Since the energy stored in an inductor is a function of its current, the current through an inductor cannot change suddenly.

$$\because v = L \times \frac{di}{dt},\ i(0+) - i(0-) = \frac{1}{L} \cdot \int_{0-}^{0+} v.dt \qquad (2.8)$$

$$\therefore i(0+) - i(0-) = 0, \text{ unless } v(0) = \infty \qquad (2.9)$$

As sources have a finite voltage, inductor current cannot change suddenly. Conversely, it means that the derivative of inductor current is practically continuous. In the case of inductors coupled magnetically with each other, the flux linkage associated with the coupled inductors cannot change instantaneously.

2.5 CAPACITOR

A capacitor functions normally as a linear element within its voltage and frequency rating. Charge held by a linear capacitor varies proportionately or linearly with its voltage, where the constant of proportionality is its capacitance. For a capacitor,

$$q = C \times v \qquad (2.10)$$

In the equation above, **v** is the voltage across the capacitor at any instant, ***C*** is the capacitance in farads and ***q*** is the charge held by the capacitor in coulombs. If a capacitor is assumed to be linear, its capacitance is independent of its voltage and is dependent on its geometry and the dielectric used. In a linear capacitor, the charge held by the capacitor and the voltage it sustains vary linearly, till the voltage limit of capacitor is reached. If a capacitor is charged beyond its voltage limit, it breaks down.

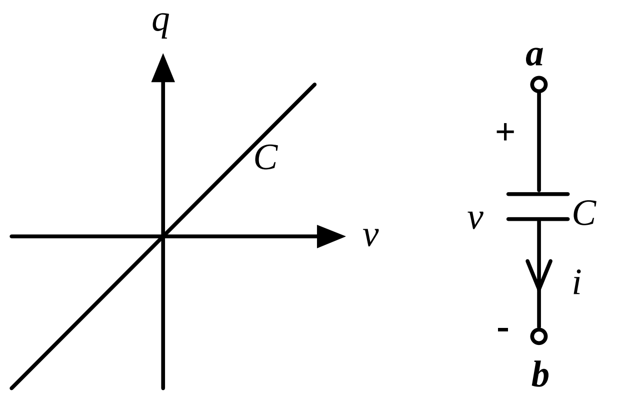
By definition, the rate of change of charge is current. Then in a capacitor,

$$i(t) = \frac{dq}{dt} = \frac{d(Cv)}{dt} = C \times \frac{dv(t)}{dt} \qquad (2.11)$$

$$\therefore v(t) = \frac{1}{C} \cdot \int_0^t i(t) \cdot dt + v(0) \qquad (2.12)$$

When capacitor voltage is evaluated from its current, it is necessary to take into account the initial capacitor voltage, which is the constant that results from the process of integration.

The *q*-*v* relationship in a capacitor is as shown in Fig. 2.4, whereas the *v*-*i* relationship in a capacitor is as shown in Fig. 2.5. If a triangular voltage signal is impressed across a capacitor, capacitor current is a square-wave signal, since it is proportional to the derivative of its voltage. When the voltage across a capacitor is rising, it is getting charged and the capacitor current is positive since the time-rate of its voltage change is positive. In simple words, the capacitor current is positive when its voltage is rising. When the capacitor voltage falls, it is discharging its energy and then its current is negative.



Symbol of a capacitor and its q-v characteristic

Fig. 2.4

The voltage across a capacitor cannot change suddenly since the capacitor current has to be infinite to bring about a sudden change in its voltage. We have that

$$\because i = C \times \frac{dv}{dt},\ v(0+) - v(0-) = \frac{1}{C} \cdot \int_{0-}^{0+} i.dt \qquad (2.13)$$

$$\therefore v(0+) - v(0-) = 0, \text{ unless } i(0) = \infty \qquad (2.14)$$

Energy gets stored in a capacitor due to the voltage it sustains. If the voltage across a capacitor rises from zero to *V* in *T* seconds, the energy, *W* stored is:

$$W = \int_0^T \left[v(t) \cdot i(t) \right] \cdot dt \qquad (2.15)$$

Substituting for *i*(*t*) from equation (2.11), we get that

$$W = \int_0^T \left[C \frac{dv}{dt} \cdot \times v(t) \right] \cdot dt = \int_0^V \left[C \times v(t) \right] \cdot dv = \frac{1}{2} C \times V^2 \qquad (2.16)$$

Capacitor voltage cannot change instantaneously primarily because the energy stored is a function of its voltage. A capacitor is also classified as a dynamic element, because it is an element which can store energy.

Analogy between inductance and capacitance is illustrated by Table 2.1. Based on analogy, it is possible form dual of a given circuit. There will be further discussion on duality of circuits in subsequent chapters to follow.

Table 2.1: Analogy Between Inductance and Capacitor

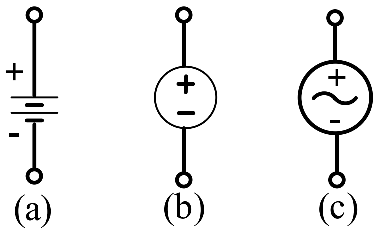
PROPERTY	**INDUCTANCE**	**CAPACITANCE**
Linearity	$\lambda = L \cdot i$	$q = C \cdot v$
Rate of change	$v(t) = \frac{d\lambda}{dt}$	$i(t) = \frac{dq}{dt}$
Value	$L = \frac{\lambda}{i} = \frac{d\lambda}{di}$	$C = \frac{q}{v} = \frac{dq}{dv}$
Energy stored	$W = \frac{1}{2} L i^2$	$W = \frac{1}{2} C v^2$

2.6 SOURCES

A source is used to excite a circuit and it can be a voltage source or a current source. It can be an independent or a dependent source. It may be a dc source or an ac source. It can be either periodic either non-periodic.

Examples of a voltage source are a battery, the utility supply and a generator. Current sources are not as common as voltage sources. It is possible to classify a welding transformer as a current source, since a welding transformer can often be set to deliver a pre-set value of current.

The symbols of some ideal voltage sources are shown in Fig. 2.5. An ideal voltage source has zero internal resistance. It means that an ideal voltage can supply any current with no drop in its terminal voltage. The terminal voltage of an ideal voltage source remains constant irrespective of value of source current. The v-i characteristic of an ideal voltage source is presented in Fig. 2.6. It can be recalled that according to Ohm's Law, resistance is the ratio of change in voltage for a given change in current. Since the terminal voltage of an ideal voltage source does not drop at any current, it can be inferred that the internal resistance should be zero. As a result, the slope of v-i characteristic of an ideal voltage source is zero.



a. Battery
b. DC Source
c. AC Source

Fig. 2.5: Voltage Sources and their Symbols

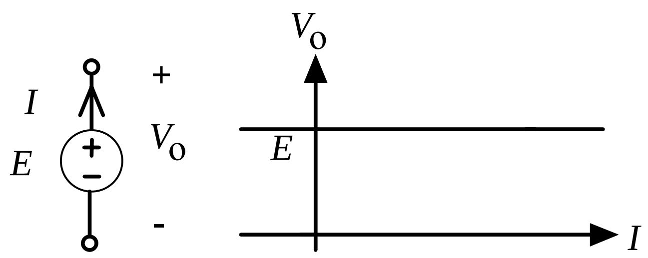


Fig.2.6: v-i characteristic of an ideal voltage source

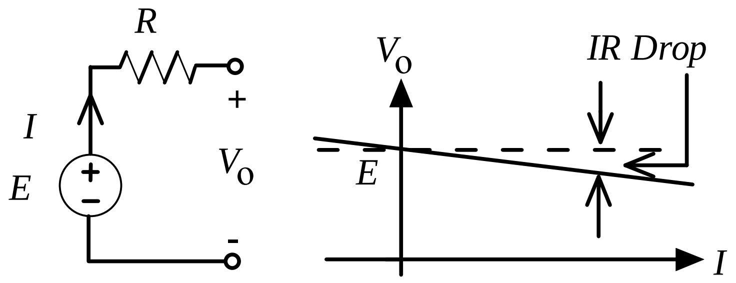
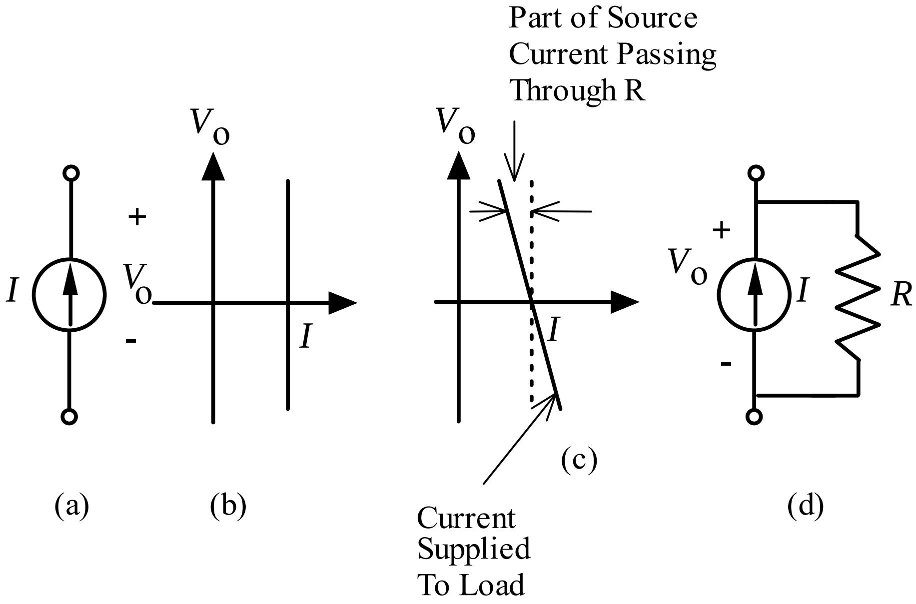


Fig.2.7: v-i characteristic of a non-ideal voltage source

A non-ideal voltage source has an internal resistance connected in series to it. This internal resistance, called as the source resistance, leads to a drop in terminal voltage as the source current increases. As the source current increases, the voltage drop across the source resistance increases and the terminal voltage of the source drops slightly. For most practical sources, the source resistance is small compared with the load resistance and the drop across the source resistance is only a small fraction of the open-circuit source voltage. The slope of the *v-i* characteristic reflects the internal resistance of the non-ideal source. The short-circuit current of a non-ideal voltage source equals *E/R*, where *E* is the open-circuit voltage of the source and *R* is its internal resistance.

An ideal current source and its *v-i* characteristic of a current source are presented in Figs. 2.8(a).and 2.8(b). A voltage source supplies any current at a constant voltage, whereas a current source supplies a constant current at any voltage. If a voltage source is short-circuited at its terminals, the short-circuit current for an ideal source can be infinite and it is limited by its internal resistance when the source is not ideal. The high short-circuit current can damage the voltage source and in general short-circuiting a voltage source should be avoided. On the other hand, an ideal current source can get damaged if it is left open-circuited with no load connected to it. When there is no load connected to the current source, it tries to maintain the same current through an open circuit. Since an open circuit can be treated as an infinite resistor, the voltage across an ideal current source can be infinite.



(a) Ideal Current Source
(b) v-i characteristic of Ideal Current Source
(c) v-i characteristic of non-ideal Current Source
(d) Non-ideal Current Source with Internal Resistance

Fig. 2.8: Current Source: Symbol and its v-i characteristic

A non-ideal current source can be represented, as shown in Fig. 2.8(d) and its v-i characteristic in Fig. 2.8(c). The internal resistance connected across the current source is relatively of high value, compared to the load resistance that would be connected to the source under normal conditions. The open-circuit voltage of a non-ideal current source is given by the product of its source current and its internal resistance. As the terminal voltage across the current source rises, the current drawn by the source resistance increases and the net current supplied to the load falls. The ***v-i*** characteristic in Fig. 2.8(c) shows that the current supplied to the load gets reduced as the voltage across the source increases.

Remember that the source resistance of a current source is high and that of the voltage source is small. We know that an ideal current source supplies a constant current at any voltage. It means that for a large change in voltage across the current source, the change in its current is zero. Based on Ohm's Law, it can be stated that an ideal current source has infinite source resistance. The vertical *v-i* characteristic of the ideal current source has infinite slope, indicating that its internal resistance is infinity. On the other hand, the source resistance is zero for an ideal voltage source.

2.7 CLASSIFICATION

This section presents more information on classification of elements. An element can be classified to be in several ways.

1. Active or passive
2. Linear or nonlinear
3. Lumped or distributed
4. Fixed or time-varying
5. Unilateral or bilateral.

A network element is classified as active or passive based on the net power absorbed or delivered by it. If a network element can supply positive average power (absorb negative energy), then it is an active element, where the average may be taken over one cycle or over a long time interval. An element that cannot supply positive average power is a passive element. A passive element may absorb either no positive power or some positive power.

Semiconductor devices such as a BJT(bipolar junction transistor) and a diode are classified as active devices. A BJT can be modelled as a dependent current source and hence, when an amplifier circuit with a BJT is analyzed, it is treated as a dependent source. A diode such as a photo diode can act as a source and hence the semiconductor devices in general are classified to be active elements.

In this text, we deal with only lumped elements. A discrete resistor, or an inductor or a capacitor is a lumped element. On the other hand, transmission lines carrying utility power have distributed inductance and capacitance. Analysis of circuit with distributed parameters is more difficult and is outside the scope of this text. In reality, most elements age, but nonetheless, this text deals with fixed elements, meaning that component values do not change with time. Such elements are time-invariant.

An element, such as a resistor which has the same signal processing ability in either direction, is a bilateral element. Normally the passive elements are bilateral. On the other hand, a device such as a diode conducts only in one direction and such an element is classified as an unilateral element.

2.8 DEPENDENT SOURCES

Sources can be classified to be either independent or dependent sources. In the section above, an independent source such as a battery has been briefly described. In this section, a dependent source is described.

A dependent source can be either a voltage source or a current source. In a dependent source, the output voltage or current may be controlled by another voltage or another current. For example, the simple small-signal model of a BJT is shown below. It can be seen from Fig. 2.9 that the BJT acts as a current source. Its output current is labeled as $\boldsymbol{i}_c$. When it is expressed as follows,

$$i_c = \beta . i_b \quad (2.17)$$

the BJT behaves as a current-controlled current source. That is, its output current is dependent on another current, labeled as $\boldsymbol{i}_b$ here. The output current of the BJT can also be expressed as given below.

$i_c = g_m.v_\pi \qquad (2.18)$

In this case, the BJT behaves as a voltage-controlled current source. That is, its output current is dependent on another signal, labeled as voltage v_π here.

In general, a dependent source can be one of the four following types:

1. Voltage-controlled Voltage Source,
2. Voltage-controlled Current Source,
3. Current-controlled Voltage Source,
4. Current-controlled Current Source.

It is possible that a dependent source may have an output resistance. The output resistance of a dependent voltage source is shown connected in series with that source, whereas the output resistance of a dependent current source is assumed to be connected in parallel with that source.

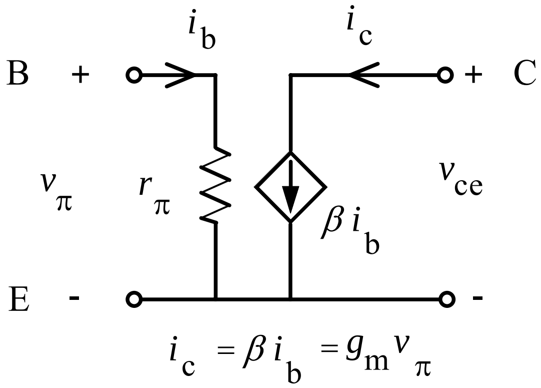


Fig. 2.9: Small Signal Model of a BJT

2.9 NON-IDEAL PASSIVE ELEMENTS

Ideally passive elements should be linear. In reality, all passive elements exhibit some non-linearity. For example, the resistance of wire-wound resistors increases as its temperature rises. Air-core inductors are almost linear, whereas an inductor with an iron-core or a ferrite-core is nonlinear, since the core tends to get saturated as flux density increases above a certain level. In addition, the relative permeability varies as a function of magnetic field strength and hence the inductance varies as a function of the current through the inductor. A capacitor is predominantly linear in its operating range. A capacitor such as an electrolytic capacitor is polarized and it can sustain voltage only in one direction. The model of an electrolytic capacitor at high frequency may include resistor and an inductor. The resistor reflects the losses in the capacitor and the inductor the inductance of the leads and the coil structure of the foil used inside the capacitor.
It needs to be mentioned that non-polarized low voltage electrolytic capacitors are available nowadays. Even though the passive elements are not strictly linear, this text treat the passive elements as linear.

2.10 WORKED EXAMPLES

Example 2.1:

Voltage, $v(t)$ connected to a 5 Ω resistor shown in Fig. 2.10 is defined as:

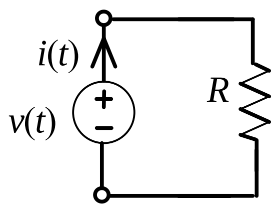


Fig. 2.10: A circuit

$$v(t)=10\cdot e^{-t}\cdot\sin(2t)\qquad \text{for } t\geq 0\text{ s}$$

Determine the energy absorbed by the resistor over $0\leq t<\infty$

Solution:

$$v(t)=10\cdot e^{-t}\cdot\sin(2t)\qquad \text{for } t\geq 0\text{ s}$$

$$i(t)=\frac{v(t)}{R}=2\cdot e^{-t}\cdot\sin(2t)$$

$$W=\int_0^\infty\left[v(t)\cdot i(t)\right]\cdot dt=20\int_0^\infty e^{-2t}\cdot\left(\sin^2(2t)\right)\cdot dt$$

$$\because \cos(2\theta)=1-2\cdot\sin^2(\theta),$$

$$W=10\cdot\int_0^\infty e^{-2t}\cdot\left[1-\cos(4t)\right]\cdot dt$$

We get the result by integrating by parts.

$$W=10\cdot\left[\frac{e^{-2t}}{-2}\right]_0^\infty-10\cdot\int_0^\infty e^{-2t}\cdot\cos(4t)\cdot dt=5-10\cdot\int_0^\infty e^{-2t}\cdot\cos(4t)\cdot dt$$

$$\int_0^\infty e^{-2t}\cdot\cos(4t)\cdot dt=\left[\frac{e^{-2t}}{-2}\cdot\cos(4t)\right]_0^\infty-2\cdot\int_0^\infty e^{-2t}\cdot\sin(4t)\cdot dt$$

$$=\frac{1}{2}+\left[\frac{e^{-2t}}{2}\cdot\sin(4t)\right]_0^\infty-4\cdot\int_0^\infty e^{-2t}\cdot\cos(4t)\cdot dt$$

$$\therefore\int_0^\infty e^{-2t}\cdot\cos(4t)\cdot dt=\frac{1}{10}.\qquad \therefore W=4\text{ J}$$

Example 2.2:

For the circuit in Fig. 2.10, $v(t)=100\cdot\cos(\omega t)$, $R=10\,\Omega$

Find power, *P* supplied to the resistor.

Solution:

Power is defined energy per second. Given an ac input, power is then the product of energy consumed per cycle and the frequency. If the frequency of source is *f* and the period of a cycle is *T*, then

$$f\cdot T=1$$

In addition, the value of ωt varies from 0 radians to 2π radians in a cycle. It means that

$$\omega \cdot T = 2\pi$$

where ω is the angular radians per second, and its value is the product 2π and frequency, f since the variation in angle is 2π radians per cycle. That is,

The voltage across resistor is $v(t)$. Then power, P is obtained as follows:

$$P = f \cdot \int_0^T \left[v(t) \cdot i(t)\right] \cdot dt = \frac{1}{T} \cdot \int_0^T \left[v(t) \cdot i(t)\right] \cdot dt, \quad \because f \cdot T = 1$$

Let

$$\omega \cdot T = 2\pi f T = 2\pi, \quad \because fT = 1$$

$$E = 100 \text{ V}, \quad \omega t = \theta. \quad \therefore \quad dt = \frac{d\theta}{\omega}.$$

Since $\omega \cdot T = 2\pi$,

$$P = \frac{1}{2\pi} \cdot \int_0^{2\pi} \left[v(\theta) \cdot i(\theta)\right] \cdot d\theta = \frac{E^2}{2\pi R} \cdot \int_0^{2\pi} \left[\cos^2(\theta)\right] \cdot d\theta$$

$$= \frac{E^2}{2\pi R} \cdot \int_0^{2\pi} \left[\frac{1 + \cos(2\theta)}{2}\right] \cdot d\theta = \frac{E^2}{2R} = 500 \text{ Watts}$$

2.11 SUMMARY

The aim of this chapter has been to provide a brief introduction to elements used in circuit analysis. The linear relationship in a passive element has been described and the nature of ideal and non-ideal sources has been illustrated. The next chapter introduces Kirchoff's laws, which are fundamental to circuit analysis.

Exercise Problems:

E2.1: For the circuit in Fig. 2.10, the waveform of voltage is as shown in Fig. 2.11. Given that *R* is 10 Ω, find power *P* supplied to the resistor. (A square-wave input)

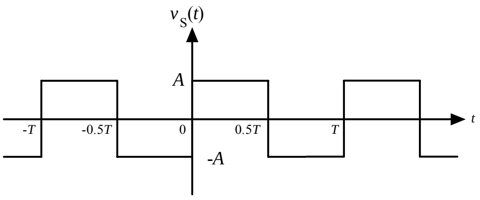


Fig. 2.11: A square-wave input signal

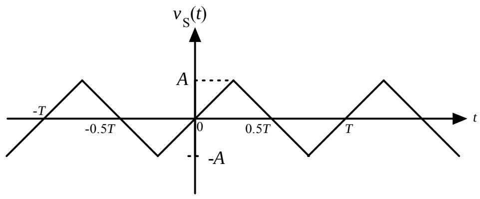


Fig. 2.12: A triangular-wave input signal

E2.2: For the circuit in Fig. 2.10, the waveform of voltage is as shown in Fig. 2.12. Given that *R* is 10 Ω, find power *P* supplied to the resistor.

E.2.3: A 2 H inductor has an initial energy of 4 J at $t = 0$ second.. Its energy rises to 25 J in 3 seconds, with its current increasing linearly. After $t > 3$ s, the inductor current remains steady. Plot the waveform of inductor current and its voltage.

E.2.4: A 3 F capacitor has zero initial energy at $t = 0$ second.. Its energy rises to 1536 J in 4 seconds, with its voltage increasing square of time. After $t > 4$ s, the capacitor voltage remains steady. Get an expression for the capacitor voltage. Plot the waveform of capacitor voltage and its current.