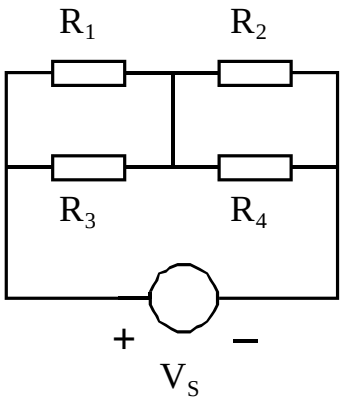
SOLUTIONS TO PROBLEMS: Chapter 4: Problem 1

Solution to Problem No. 1



$V_S = 117$ V DC
$R_1 = 9\ \Omega$
$R_2 = 5\ \Omega$
$R_3 = 12\ \Omega$
$R_4 = 9\ \Omega$

Fig. 1

Question 1:

For the circuit in Fig .1, find the current through each of the resistors and the power absorbed by each resistor. Verify the result using Tellegen's theorem.

Solution:

It is preferable to find the total current supplied by the source first. Then currents through the resistors can be determined using current division rule. It is necessary to determine the equivalent resistance seen by the source first. Let it be R_{eq}.

$R_1 := 9\ \Omega$ $R_2 := 5\ \Omega$

$R_3 := 12\ \Omega$ $R_4 := 9\ \Omega$

$$R_{eq} := \frac{R_1 \cdot R_3}{R_1 + R_3} + \frac{R_2 \cdot R_4}{R_2 + R_4} \qquad R_{eq} = 8.357 \quad \Omega$$

$V_S := 117 \quad V$

Current supplied by source is: $I_S := \frac{V_S}{R_{eq}} \qquad I_S = 14 \quad A$

Using current division rule:

$$I_{R1} := I_S \cdot \frac{R_3}{R_3 + R_1} \qquad I_{R1} = 8 \quad A$$

$$I_{R3} := I_S \cdot \frac{R_1}{R_3 + R_1} \qquad I_{R3} = 6 \quad A$$

$$I_{R2} := I_S \cdot \frac{R_4}{R_4 + R_2} \qquad I_{R2} = 9 \quad A$$

$$I_{R4} := I_S \cdot \frac{R_2}{R_4 + R_2} \qquad I_{R4} = 5 \quad A$$

Verification using Tellegen's theorem.

Determine the power supplied by the source.
Determine the power dissipated in the resistors.
These two results should equal each other for power balance.
According to passive sign convention power supplied by the source is negative, since current flows out of its positive terminal. Power consumed by each resistor is positive, since current flows into the positive terminal of any of the resistors. In that case, both voltage and current associated with a resistor have the same sign, according to the passive sign convention. Hence power dissipated by a resistor is positive. Tellegn's theorem states that the algebraic sum of power of all elements in a circuit is zero.

$P_S := -V_S \cdot I_S$ $\qquad P_S = -1.638 \times 10^3$ W

$P_R := I_{R1}^2 \cdot R_1 + I_{R2}^2 \cdot R_2 + I_{R3}^2 \cdot R_3 + I_{R4}^2 \cdot R_4$

$P_R = 1.638 \times 10^3$ W

$P_S + P_R = 0$

E4.2:

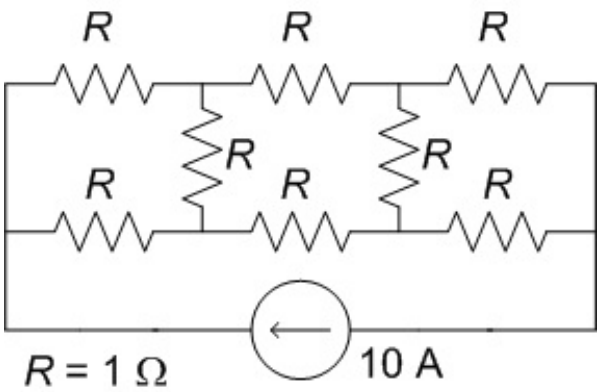


Fig. : EP2

For the circuit in Fig. EP2, obtain the voltage across the source. Find also currents through each of the resistors.

Solution:

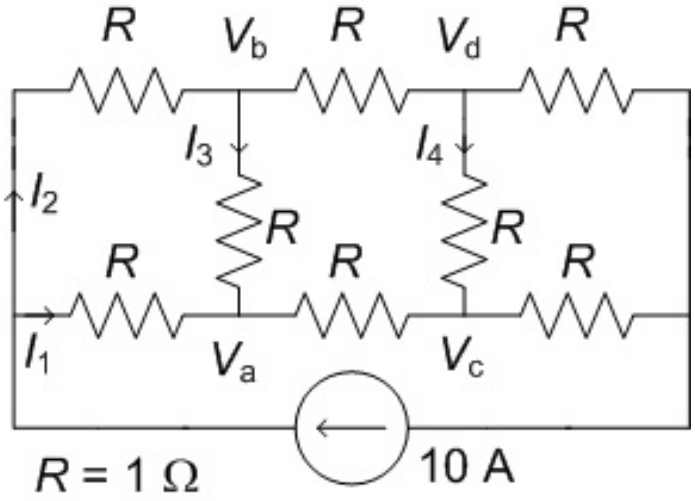


Fig. : Solution to EP2

Let voltage across source be V_S. By symmetry,

$$V_a = V_b \quad \therefore I_3 = 0$$
$$V_c = V_d \quad \therefore I_4 = 0$$
$$\therefore I_1 = I_2 = 5\ \text{A}$$
$$V_S = 3 \times I_1 = 15\ \text{V}$$

E4.3:

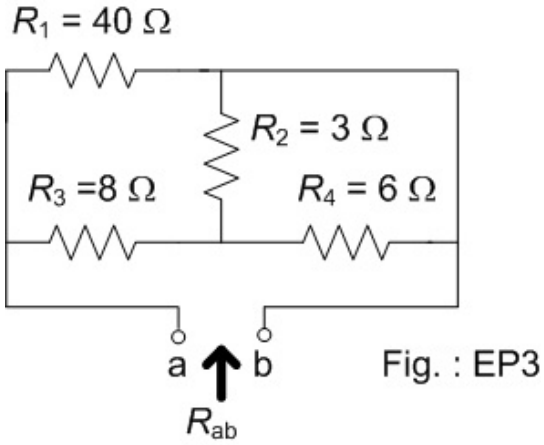


Fig. : EP3

For the circuit in Fig .EP3, find the value of resistance, $\boldsymbol{R}_{ab}$.

Solution:

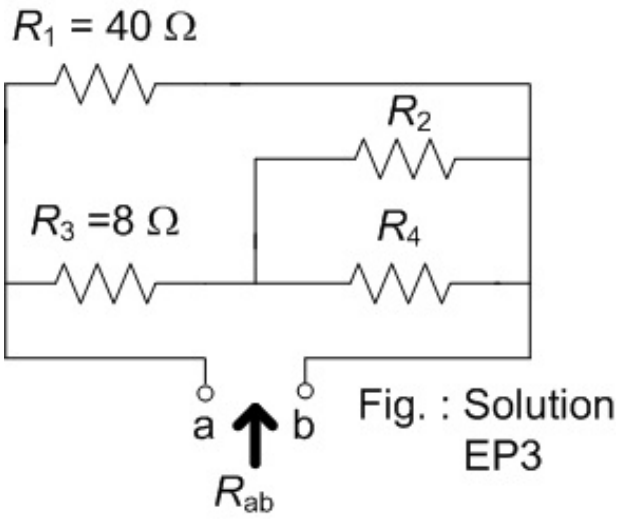


Fig. : Solution EP3

Redraw the sketch as shown above. Then

$$R_{ab} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_3 + \frac{R_2 \times R_4}{R_2 + R_4}}} = \frac{1}{\frac{1}{40} + \frac{1}{8 + \frac{18}{9}}} = 8\ \Omega$$

E4.4:

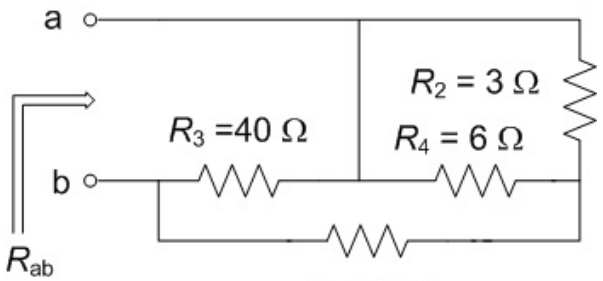


Fig. : EP4

For the circuit in Fig .EP4, find the value of resistance, $\boldsymbol{R}_{ab}$.

Solution:

The same as for the previous problem.

E4.5:

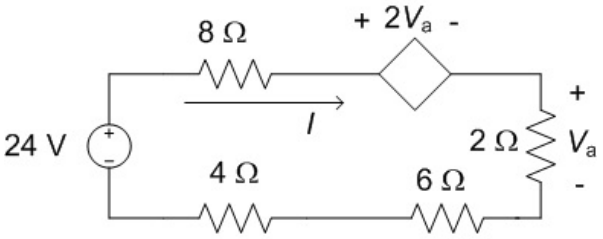


Fig. : EP5

E4.5: For the circuit in Fig. EP5, find the loop current, I, and the voltage, $\boldsymbol{V}_a$.

Solution:

If the drop across 2 is V_a, the drops across other resistors is proportionate to their resistance. Hence

$$10V_a - 2V_a = 24 \text{ V}.$$
$$\therefore V_a = 3 \text{ V}, \; I = 1.5 \text{ A}$$