SERIES-PARALLEL CIRCUITS

This chapter describes the elementary connection patterns for resistors, capacitors and inductors in a circuit.

SERIES CONNECTION OF RESISTORS

A circuit with three resistors in series is shown in Fig. 4.1. Using Ohms' law and Kirchoff's voltage law, we get that

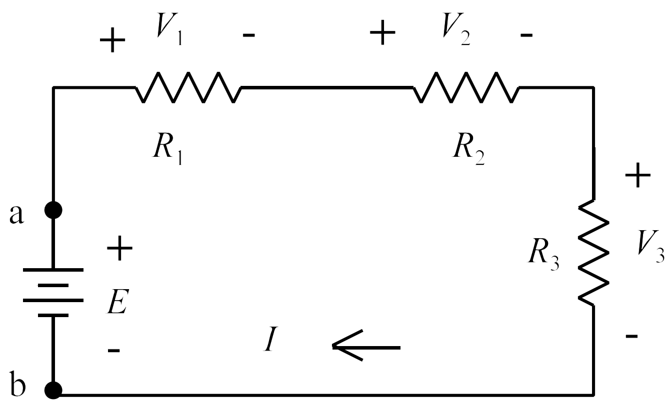

Fig. 4.1: Series Connection of Resistors

$$E = I.(R_1 + R_2 + R_3) \quad (4.1)$$

The above equation can be expressed as follows:

$$E = I.R_{eq} \quad \text{where} \quad R_{eq} = (R_1 + R_2 + R_3) \quad (4.2)$$

It is seen that the three resistors connected in series can be replaced by a single equivalent resistor and its resistance equals the sum of the three resistances. This rule can be extended to any number of resistors in series.

From Fig. 4.1, we find also that $\dfrac{V_1}{E} = \dfrac{I.R_1}{I.R_{eq}} = \dfrac{R_1}{R_{eq}}$ (4.3)

From equation (4.3), we can generalize that **when resistors are connected in series, voltage division among them is proportionate to the resistance**, the scale factor relating the voltage to the resistance being the current through it. That is,

$$\frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V_3}{R_3} = \frac{E}{R_{eq}} \quad (4.4)$$

PARALLEL CONNECTION OF RESISTORS

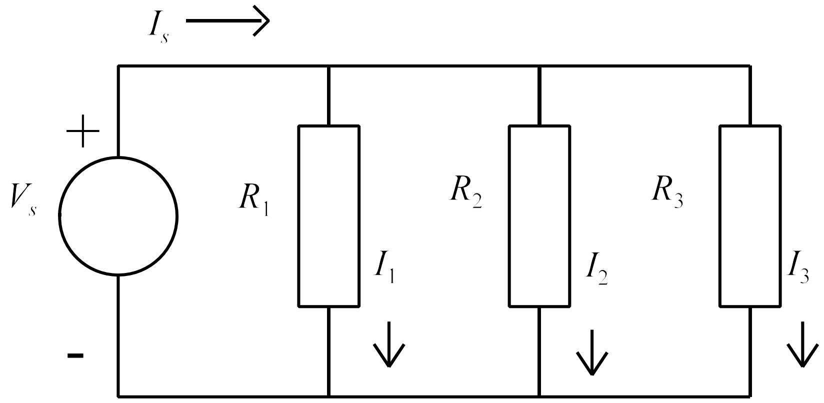

Fig. 4.2: Parallel Connection of Resistors

A circuit with three resistors in parallel is shown in Fig. 4.2. Using KCL, we obtain that

$$I_S = I_1 + I_2 + I_3 \qquad (4.5)$$

Let the equivalent resistance of three resistors in parallel be R_{eq}. If R_{eq} is connected across source V_s shown in Fig. 4.2, it would draw a current equaling I_S. Then equation (4.5) can be presented as follows:

$$\frac{V_S}{R_{eq}} = \frac{V_S}{R_1} + \frac{V_S}{R_2} + \frac{V_S}{R_3} \qquad (4.6)$$

On canceling the common numerator term, we obtain that

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (4.7)$$

The reciprocal of resistance is conductance. Usually the letter *G* is used to denote conductance and its unit is siemens(S). Let

$$R_{eq} = \frac{1}{G_{eq}},\ R_1 = \frac{1}{G_1},\ R_2 = \frac{1}{G_2},\ \text{and}\ \ R_3 = \frac{1}{G_3} \qquad (4.8)$$

Then $\quad G_{eq} = (G_1 + G_2 + G_3) \quad (4.9)$

That is, $\quad R_{eq} = \dfrac{1}{G_{eq}} = \dfrac{1}{G_1 + G_2 + G_3} = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} \qquad (4.10)$

When resistors are in parallel, the equivalent resistance can be found out as shown in equation (4.10). This method can be extended to any number of resistors in parallel.

The voltage across each of the resistors in Fig. 4.2 is the same. That is,

$$V_S = I_S \times R_{eq} = I_1.R_1 = I_2.R_2 = I_3.R_3 \quad (4.11)$$

Then $$V_S = \frac{I_S}{G_{eq}} = \frac{I_1}{G_1} = \frac{I_2}{G_2} = \frac{I_3}{G_3} \quad (4.12)$$

It is seen from equation (4.12) that **when resistors are in parallel, current division is proportionate to conductance** and the scale factor relating current to conductance is the source voltage.

COMBINED CONNECTION

Many circuits have combined connections. The circuit in Fig. 4.3 has a combined connection. It is of interest to find out the resistance seen by the source. To calculate the resistance seen by the source, the circuit in Fig. 4.3 can be re-drawn as shown in Fig. 4.4, where R_p equals R_2 and R_3 connected in parallel. Then

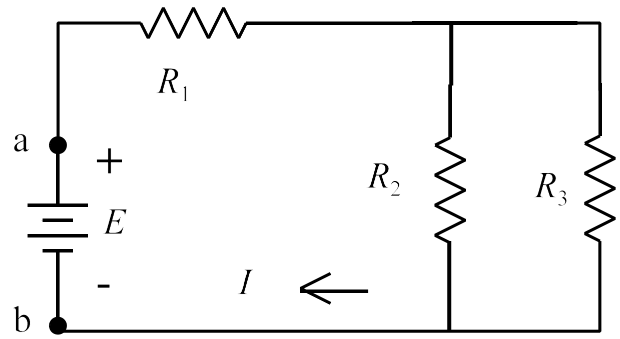

Fig. 4.3: **Series-Parallel Connection**

$$\frac{E}{I} = R_1 + R_p = R_1 + \frac{R_2 \times R_3}{R_2 + R_3}$$

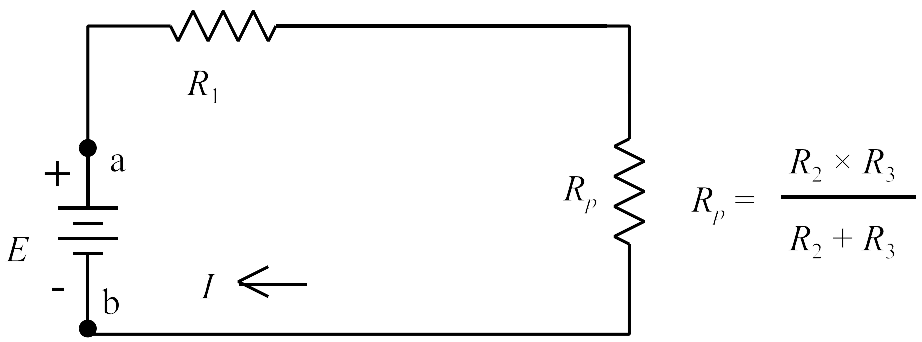

Fig. 4.4: **Circuit in Fig. 4.3 Simplified**

SERIES CONNECTION OF CAPACITORS

Capacitors can also be connected in series or parallel, in order to meet the performance requirements. In Fig. 4.5, three capacitors are in connected in series. Since they are in series, the current through them is the same, and each capacitor holds the same charge. From Fig. 4.5,

$$E = V_1 + V_2 + V_3 \quad (4.13)$$

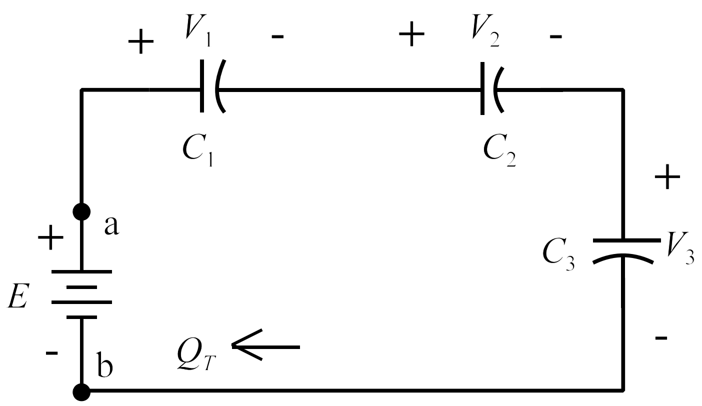

Fig. 4.5: Series Connection of Capacitors

Since the charge held by a capacitor equals the product of the voltage across it and its capacitance,

$$E = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \quad (4.14)$$

It can be stated that an equivalent capacitance, C_{eq} has the charge Q and a voltage of E across it. Then

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}, \text{ or } \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad (4.15)$$

By connecting capacitors in series, the resultant equivalent capacitance obtained is smaller in value, but it can sustain a much higher voltage across it. Hence capacitors are connected in series in order to get an equivalent capacitor of a much higher voltage rating. In practice, in order to ensure that the capacitors share the voltage in the desired proportion, a relatively large valued resistor is connected across each capacitor. The resistor connected across each capacitor has to be large in order to reduce the power loss that can occur.

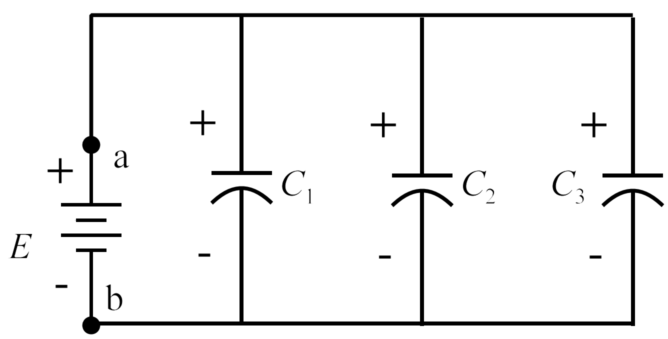

Fig. 4.6: Parallel Connection of Capacitors

PARALLEL CONNECTION OF CAPACITORS

The circuit in Fig. 4.6 shows three capacitors in parallel. The total charge held by the three capacitors is the sum of the charges held by those capacitors. Let the total charge be Q_T. Then

$$Q_T = Q_1 + Q_2 + Q_3 = \left(E \times C_1\right) + \left(E \times C_2\right) + \left(E \times C_3\right) \quad (4.16)$$

It can be said that an equivalent capacitance C_{eq} has charge Q_T, with its voltage equal to E. Then $Q_T = C_{eq} \times E$. That is,

$$C_{eq} = C_1 + C_2 + C_3 \quad (4.17)$$

Capacitors are connected in parallel in order to get an equivalent capacitance of a much higher value. It is possible that a single capacitor with the desired capacitance may not be available and hence capacitors may be connected in parallel. In some power supply filter circuits, it may be desirable to connect capacitors in parallel in order to ensure that these capacitors have the adequate ripple current rating. For example, a 1000 μF, 100 V may have a ripple current rating of 5 A, whereas two 500 μF, 100 V may have a combined ripple current rating of 6 A and hence it may be preferable to connect them in parallel.

SERIES CONNECTION OF INDUCTORS

It is possible to connect inductors in series, as shown in Fig. 4.7. Assume that the source voltage varies as a function of time. Then the current through the inductors would change. Since they are connected in series, the rate of change of current is the same for all the inductors. Then

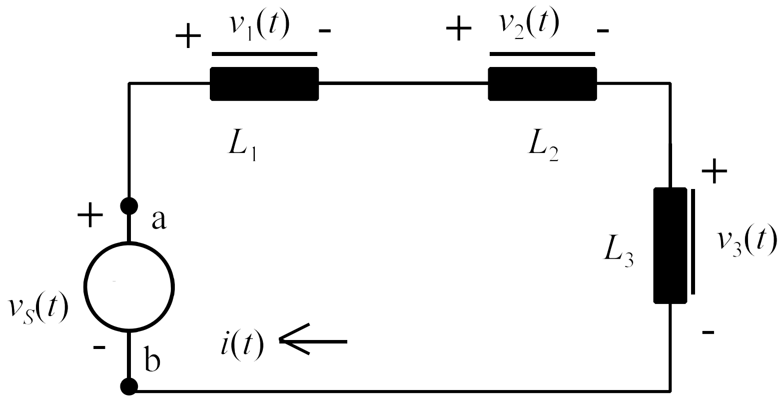

Fig. 4.7: Series Connection of Inductors

$$v_S(t) = v_1(t) + v_2(t) + v_3(t) = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt}$$

$$= \left(L_1 + L_2 + L_3\right) \frac{di}{dt} = L_{eq} \frac{di}{dt} \quad (4.18)$$

It can be seen that the three inductors in series is equivalent to one inductor, with its inductance being equal to the sum of inductances. In practice, connecting inductors in series may be a convenient method to obtain a larger inductor.

PARALLEL CONNECTION OF INDUCTORS

Three inductors are connected in parallel, as shown in Fig. 4.8. It can be seen that

$$\frac{di_S}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt} \quad (4.19)$$

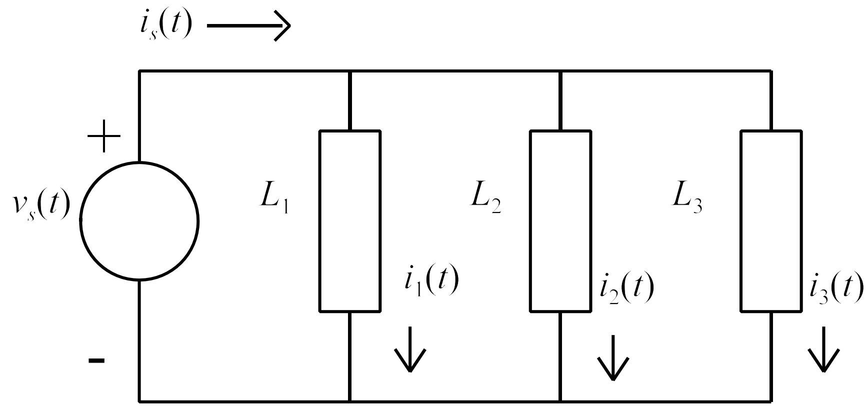

Fig. 4.8: Parallel Connection of Inductors

Let there be an equivalent inductance such that its rate of change of current equals di_S/dt when the voltage across it equals $v_S(t)$. Then

$$\frac{v_S(t)}{L_{eq}} = \frac{v_S(t)}{L_1} + \frac{v_S(t)}{L_2} + \frac{v_S(t)}{L_3} \text{ , or } \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \quad (4.20)$$

SUMMARY

This page has illustrated how resistors,inductos and capacitors in series or parallel can be replaced by an equivalent component. It is also shown how the resistors in series divide the voltage applied to the string of resistors in series. Current division among resistors in parallel has also been explained. Next chapter describes the theorems used in circuit analysis.

Exercise Problems:

E4.1:

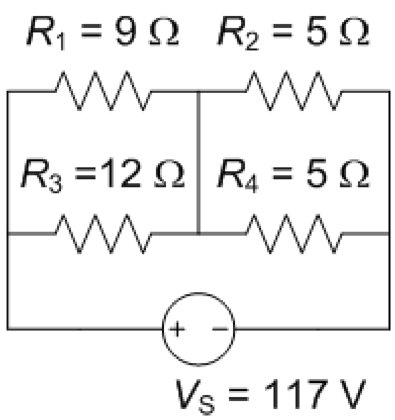

Fig. : EP1

For the circuit in Fig. EP1, obtain the current drawn from the source. Find also currents through each of the resistors.

Find the power delivered by the source, and the power consumed by each of the resistors. Verify that the algebraic sum of power in all elements is zero.

E4.2:

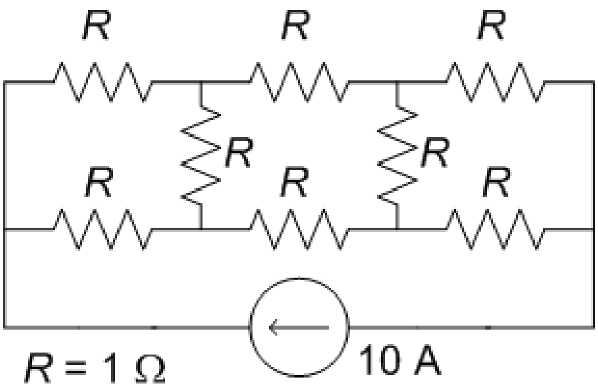

Fig. : EP2

For the circuit in Fig. EP2, obtain the voltage across the source. Find also currents through each of the resistors.

E4.3:

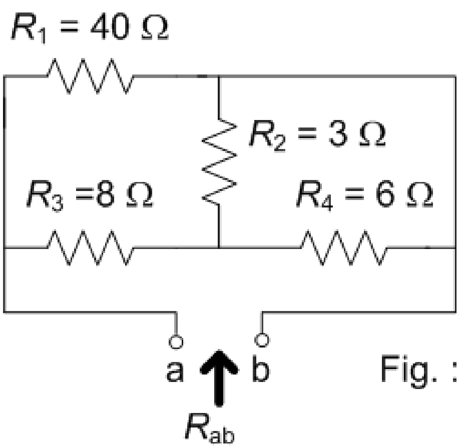

Fig. : EP3

For the circuit in Fig .EP3, find the value of resistance, $\boldsymbol{R}_{ab}$.

E4.4:

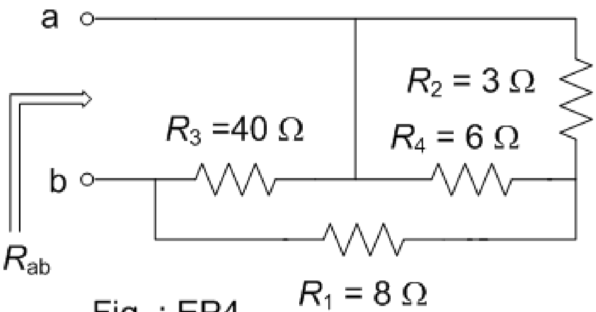

Fig. : EP4

For the circuit in Fig .EP4, find the value of resistance, $\boldsymbol{R}_{ab}$.

E4.5:

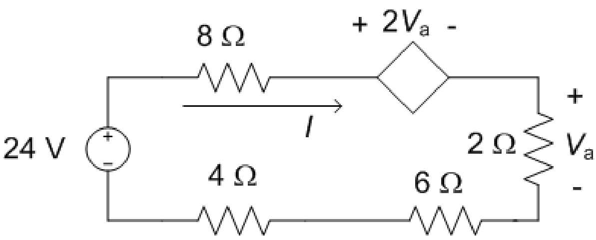

Fig. : EP5

E4.5: For the circuit in Fig. EP5, find the loop current, *I*, and the voltage, $\boldsymbol{V}_a$.